

Co-primary inter-operator spectrum sharing over a limited spectrum pool using repeated games

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Abstract—We consider two small cell operators deployed in the same geographical area, sharing spectrum resources from a common pool. A method is investigated to coordinate the utilization of the spectrum pool without monetary transactions and without revealing operator-specific information to other parties. For this, we construct a protocol based on asking and receiving *spectrum usage favors* by the operators, and keeping a book of the favors. A spectrum usage favor is exchanged between the operators if one is asking for a permission to use some of the resources from the pool on an exclusive basis, and the other is willing to accept that. As a result, the proposed method does not force an operator to take action. An operator with a high load may take spectrum usage favors from an operator that has few users to serve, and it is likely to return these favors in the future to show a cooperative spirit and maintain reciprocity. We formulate the interactions between the operators as a repeated game and determine rules to decide whether to ask or grant a favor at each stage game. We illustrate that under frequent network load variations, which are expected to be prominent in small cell deployments, both operators can attain higher user rates as compared to the case of no coordination of the resource utilization.

Keywords—Co-primary spectrum sharing, repeated games, spectrum pooling.

I. INTRODUCTION

In the state-of-art mobile communication systems, a network operator possesses a spectrum license that provides exclusive transmission rights for a particular range of radio frequencies. Spectrum assignment based on dedicated licenses resolves the issues related to inter-operator interference but it also results in low spectrum utilization efficiency. Inter-operator spectrum sharing is envisioned as one of the viable approaches to achieve higher operational bandwidth efficiency and meet the increasing mobile data traffic demand in a timely manner [1].

In the limited spectrum pool (LSP) scenario, a limited number of operators share a common resource pool by relying on more flexible and adaptive prioritization policies than is currently possible with dedicated licenses [2]. Cognitive radio technologies are effective measures to resolve the sharing conflicts over the LSP under vertical spectrum sharing [3], where the lessor (owner) operator has higher legacy rights over the spectrum than the lessee operator. On the other hand, the co-primary or horizontal spectrum sharing scheme conceptualizes the case where authorized operators possess equal ownership on the spectrum being adopted [4]. However, *a priori* agreements should be made on the spectrum usage with regard to the long term share of an individual operator.

The multilateral use of shared resources in the LSP can, for instance, be achieved with channel allocation schemes originally developed for single-operator systems. These schemes

are in principle applicable to realizes inter-operator spectrum sharing, provided that the operators are willing to exchange information and cooperate honestly. Under this requirement, many spectrum sharing algorithms are available in the literature. They differ related to the domain where inter-operator interference is handled, i.e. time, frequency, and/or space.

The cooperative spectrum sharing schemes require a great deal of network information exchange among the operators, e.g., interference prices [5], channel state information [6], [7], etc., and/or employing a central entity to decide upon the resource allocation. In [5], the operator reports the inflicted aggregate interference to the spectrum controller, and on this basis, the controller awards the spectrum pool to the impacted base stations (BSs). In [6], cooperation amongst the operators is realized by broadcasting the spectrum occupancy information, allowing small cells of competitor operators to avoid interference in accessing the spectrum pool. Similarly, operators in [7] maintain channel occupancy and spectrum reservation matrices for opportunistic access to the shared pool.

Although the achievable gains in cooperative schemes are in general high, operators may be reluctant to share proprietary information with their competitors and may also have an incentive to mis-report this information. Finally, information exchange may incorporate excessive inter-operator signaling overhead. In this perspective, game theoretic non-cooperative schemes appear to be a more viable option to share spectrum. In these schemes, players make decisions independently; they may still cooperate with competitors but, the cooperation is entirely self-enforcing.

In [8], [9], operators establish cooperation and play non-zero sum games to share spectrum. However, the choice of utility function, encompassing spectrum pricing is undesirable as it penalizes increased spectrum usage. In [10], the operators enlist their preferences of partitioning the shared pool and the outcome is established based on a minimum rule. This method may not work well in scenarios with load variations, in which heavily-loaded and lightly-loaded operators will end up with the same number of orthogonal carriers from the pool. In [11], [12], operators model their interactions via repeated games. A common assumption is that they agree in advance on the spectrum allocation, e.g., at a Nash equilibrium (NE) in [11] or at an orthogonal allocation in [12] and this allocation is maintained under the threat of punishment. Auction-based sharing techniques have been discussed in [13], [14], in which operators bid competitively for spectrum access through a spectrum broker. However, operators may be hesitant in adopting market-driven sharing schemes as they may not want to touch their revenue model.

In this paper, we consider spectrum sharing in a setting

where no radio access network (RAN) information is revealed to other operators. We assume that operators are not willing to monetize spectrum use, keeping spectrum sharing on the RAN level. Unlike one-shot games, the proposed scheme takes into account also the history of previous interactions between the operators and entails the benefits of reciprocity. We illustrate that a repeated game can be set up so that both operators achieve better performance in comparison to a static spectrum allocation. Unlike the repeated game models proposed in [11], [12], we do not fix the spectrum allocation but we allow a flexible use of the LSP based on the network load and interference conditions. By employing the proposed scheme in a scenario with two operators, we are able to show that under load asymmetry, both operators can benefit as compared to a scheme where no spectrum coordination is allowed.

The remainder of the paper is organized as follows. In Section II, we present the system model. Section III formulates the repeated game for inter-operator spectrum sharing and presents the proposed mechanism for negotiating the utilization of the spectrum pool. Section IV demonstrates performance gains with the proposed scheme and finally Section V concludes the paper with a summary and areas for further work.

II. SYSTEM MODEL

For simplicity, we concentrate on a spectrum sharing scenario with two small cell operators, Operator A and Operator B . Each operator has one component carrier (CC) for dedicated usage. The operators participate also in a LSP and divide it into K CCs of equal bandwidth. The proposed spectrum sharing scheme will be used to negotiate the utilization of the K CCs in the downlink.

We consider a scenario with network load variations. At a particular time instant, the user distribution is modeled via a Poisson point process (PPP) with a mean equal to N_X users for Operator $X: X \in \{A, B\}$. Given the network state, an operator evaluates a network utility function to describe the quality-of-service (QoS) offered to its users. It is important to remark that operators need not employ same utility function nor to be aware of each other's utility function. For simplicity, we assume that both operators maintain a proportionally fair [15] utility function directly constructed based on the user rates.

$$U_X = \sum_{n=1}^{n_X} \log \left(\sum_{k=1}^{K+1} r_{n,k} \right)$$

where n_X is a particular realization of a PPP with mean N_X , $r_{n,k}$ is the transmission rate of the n -th user of Operator X on the k -th CC calculated as

$$r_{n,k} = w_{n,k} B \log_2 \left(1 + \frac{\gamma_{n,k}}{\gamma_{\text{eff}}} \right)$$

where $w_{n,k}$ is the time scheduling weight of the n -th user scheduled on the k -th CC, B is the bandwidth of a CC, $\gamma_{n,k}$ is the downlink user SINR and γ_{eff} is the SINR efficiency.

We consider downlink transmissions without power control. The downlink received signal power for the n -th user on the k -th CC is $S_{n,k}$. Also, let us denote by $I_{n,k}$ the aggregate interference level incorporating both interference from the operator's own network, and from the other operator's interfering BSs. Then, the downlink user SINR is

$$\gamma_{n,k} = \frac{S_{n,k}}{I_0 + I_{n,k}}$$

where I_0 is the power per CC of thermal noise and other

interference. Note that on the dedicated CC there is no inter-operator interference.

The scheduling weights, $w_{n,k}$ are determined to maximize the utility U_X .

$$\begin{aligned} &\text{Maximize :} && U_X. \\ &w_{n,k} \\ &\text{Subject to :} && \sum_{n=1}^{n_X} w_{n,k} = 1 \forall k \\ &&& w_{n,k} \geq 0, \forall \{n, k\}. \end{aligned}$$

In order to evaluate the effect the opponent operator has on its utility, an operator may ask its users to measure the amount of aggregate interference level they receive from the opponent. This functionality requires that the users are able to separate between their own and the other operator's generated interference. Note that this kind of functionality does not require any signaling between the two operators. It is assumed that inter-operator interference measurements are ideal.

III. COORDINATION PROTOCOL

In small cell deployments it is expected to have changing traffic and interference profiles. Small cell deployments of different operators sharing spectrum in the same geographical area can exploit these fluctuations and achieve mutual benefits by regulating the allocation of CCs. For instance, let us consider spectrum sharing between two operators with unbalanced traffic loads over a LSP. A lightly-loaded operator can satisfy its QoS with few CCs and could perhaps stop using some of the CCs from the pool. An operator that is heavily loaded at that time would not suffer from inter-operator interference on the emptied CCs and would be able to meet its QoS too. However, there should be an incentive for the lightly-loaded operator to free up some CCs.

Without going to monetary transactions, we propose to regulate the allocation of CCs by means of *spectrum usage favors* asked and granted by the operators. In a LSP, a spectrum usage favor refers to the following action – an operator asks its competitor for permission to start using a certain number of CCs from the pool on an exclusive basis. While negotiating for spectrum, the operators should agree about the default utilization of the spectrum pool. In principle, any MAC protocol could be applied in the default state. We consider that both operators utilize all the CCs of the pool. A spectrum usage favor that is exchanged between the operators necessitates a departure from the default state. The time period a spectrum favor is valid is agreed between the operators. E.g., a favor can be valid in the order of seconds, reflecting the time the network states remain unchanged. After the validity of a favor expires, the utilization of the spectrum pool falls back to the default state and the operators will begin a new round of negotiations.

Since the operators will share spectrum for a long time, an operator taken favors in the past will return them in future to show a cooperative spirit and maintain the exchange of favors with its competitor. Monetized compensations or auction schemes are not considered here. It is a non-trivial task to design such efficient mechanisms for a limited area and a limited time, and to couple operator strategies to the income model of operators. Realizing spectrum sharing in the form of favors entails the benefits of reciprocity, circumvents monetary-based spectrum sharing and enables the operators to achieve mutual benefits without revealing RAN-specific information to competitors and/or other parties. These make the considered approach similar to peering agreements in the internet.

A. One-shot game

We first consider a one-shot game where the operators A and B are modeled as myopic players. The game is strategic and non-cooperative. Each player X 's action or strategy, s_X , is to either ask a favor of $k = 1, \dots, K$ CCs denoted by (a_1, \dots, a_k) , grant a favor on $k = 1, \dots, K$ CCs denoted by (g_1, \dots, g_k) or do neither, denoted by n . Let S_X denote the set of such actions for player X .

To specify the outcome of the game, we assume that a favor is exchanged only if one player plays g_l and the other plays a_k with $l \geq k$, then the outcome is an exchange of k CCs. Otherwise, no exchange of favors occur.¹

Depending on the outcome, operators draw rewards: (i) the reward when a player takes a favor is equal to the utility gain when the interference on k CCs is eliminated, (ii) the reward when a player grants a favor is equal to the utility loss when stopping to use k CCs and (iii) the reward when a player does not ask nor grant a favor is zero. The gains and losses thus depend on the current internal state of the player in question, and are not known to the opponent.

In such a game, a final outcome is a NE $(s_A, s_B) \in S_A \times S_B$, from which no player can improve its utility by deviating unilaterally, i.e., $U_X(s_X, s_{-X}) \geq U_X(s_X^*, s_{-X})$ for every player $X = \{A, B\}$ and every alternative strategy $s_X^* \in S_X$. It is straightforward to see that the NE of the formulated one-shot game corresponds to the situation where a player always asks for a favor on K CCs to maximize its reward but never grants a favor. As a result, both operators would utilize all CCs from the pool irrespective of the network load and interference profiles.

The game formulated here differs from the power control game in [11] in that here the actions are requests of other player powers, and power control is binary, whereas in [11], the actions are power allocation profiles across frequency. Also, no information of other operator state is assumed here. The NEs of the strategic one-shot games discussed here and in [11], however, are similar; full usage of the available spectrum is the only option for a rational player.

B. Repeated game

Since the operators will share spectrum for a long time, the one-shot game described above will be played repeatedly. In a repeated game, the action of a player at a stage game depends not only on the current rewards but also in the sequence of previous rewards see, e.g., [16]. Repeated games, such as prisoner's dilemma have been well-studied and admit a rich set of equilibrium profiles [16] including various punishment strategies. The setting here is even more challenging as the game we are interested in is a *stochastic game* meaning that each player's pay-off depends on a random parameter, namely the configuration of the users at that time, and moreover the player's have *imperfect information* since they only observe their own user configuration. We thus have a *Bayesian game*. Given this, we focus on a simplified set of stationary threshold policies and characterize an equilibrium among these policies.

Negotiations of favors on a single CC were considered in [17]. Here, we extend that to a situation with multiple CCs. At each stage of the game an operator can compute its

utility gain and utility loss by asking and granting favors on k CCs for $k = 1, \dots, K$. The probability distribution function (PDF) of the utility gains when Operator X gets a favor on k CCs is denoted by f_{X,G_k} and similarly, the PDF of utility losses by f_{X,L_k} . In our system setup, the randomness is only due to the Poisson distribution of the operator's own users (and the user's corresponding channel gains). Therefore the PDFs depend only on the network state of the operator's own network. However, note that if power control and inter-cell interference coordination is employed, these distributions may depend on the state of the opponent operator's network too.

At each stage game, the Operator X first checks whether to ask for a favor on K CCs by comparing its immediate utility gain with a threshold $\theta_{X,K}$. If the utility gain is less than the threshold $\theta_{X,K}$, the operator can consider asking for a favor on $(K - 1)$ CCs instead, and so forth. We assume that an operator always asks for the largest number of CCs for which its utility gain exceeds the corresponding threshold. As a result, the probability that Operator X asks for a favor on k CCs is equal to the probability that the utility gain from taking a favor on $\{j : j = (k+1), \dots, K\}$ CCs is less than the corresponding thresholds $\theta_{X,j}$ and also, the utility gain from taking a favor on k CCs is higher than the threshold $\theta_{X,k}$

$$P_{X,k}^{\text{ask}} = \prod_{j=k+1}^K \int_0^{\theta_{X,j}} f_{X,G_j}(g_j) dg_j \int_{\theta_{X,k}}^{\infty} f_{X,G_k}(g_k) dg_k \quad (1)$$

where, to simplify the analysis, it has been assumed that the distributions of utility gains from taking favors on different number of CCs are independent.

Similarly, the Operator X grants a favor on k CCs upon being asked, if its immediate utility loss is smaller than a threshold $\lambda_{X,k}$ and it has not already requested a favor. Taking into account the fact that an operator cannot ask and grant a favor at the same stage game, the probability to grant a favor on k CCs is

$$P_{X,k}^{\text{grant}} = \prod_{j=1}^K \int_0^{\theta_{X,j}} f_{X,G_j}(g_j) dg_j \int_0^{\lambda_{X,k}} f_{X,L_k}(l_k) dl_k \quad (2)$$

where it has been assumed that the distributions of utility gains and utility losses are independent.

We assume that the networks of the operators are similar, and in symmetric relationship with each other, and do not assume a discount of favors. To get preliminary understanding on steady state behavior in such a setting, inspired by [18] we thus assume that averaged over long times, operators give and take the same amount of equally valuable favors. Hence, favors would become a rudimentary RAN-level spectrum sharing currency. Thus we have

$$\sum_{k=1}^K k P_{A,k}^{\text{ask}} P_{B,k}^{\text{grant}} = \sum_{k=1}^K k P_{B,k}^{\text{ask}} P_{A,k}^{\text{grant}} \quad (3)$$

where the left-hand side describes the average number of CCs that Operator A gets a favor on, and the right-hand side the same quantity for Operator B .

An operator can monitor the probabilities of asking and granting of the opponent and set its own decision thresholds for satisfying the constraint (3). However, there may be multiple combinations of thresholds fulfilling the constraint. We propose to identify the thresholds maximizing an excess utility \tilde{U} calculated over the NE of the one-shot game (i.e. both operators utilize simultaneously all the K CCs). The excess utility for an operator reflects its expected gain from taking a

¹In other words, here we view the requests as being for a specific number of CCs and no fewer. Alternatively, one could study a model where the asks are for up-to that number of CCs so that if $l < k$ a trade for l CCs still occurs. We leave such a model for future work.

favor penalized by its expected loss from granting a favor. In order to avoid unnecessary complexity in the notation we show how to set the decision thresholds for Operator *A*. Similarly, the decision thresholds for Operator *B* can be computed.

The excess utility for Operator *A* is

$$\tilde{U}_A = \sum_{k=1}^K (\Gamma_{A,k} - \Lambda_{A,k}) \quad (4)$$

where $\Gamma_{A,k}$ and $\Lambda_{A,k}$ are the average gain and loss in utility on *k* CCs for Operator *A* such that,

$$\Gamma_{A,k} = P_{B,k}^{\text{grant}} \prod_{j=k+1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j \int_{\theta_{A,k}}^{\infty} g_k f_{A,G_k}(g_k) dg_k,$$

$$\Lambda_{A,k} = P_{B,k}^{\text{ask}} \int_0^{\lambda_{A,k}} l_k f_{A,L_k}(l_k) dl_k \prod_{j=1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j.$$

The optimization problem for identifying the decision thresholds is

$$\begin{aligned} & \text{Maximize : } \tilde{U}_A, \\ & \theta_{A,k}, \lambda_{A,k} \forall k \\ & \text{Subject to : } \text{Eq. (3)}. \end{aligned} \quad (5)$$

In order to solve this optimization problem, we construct the Lagrangian function and solve the system of first-order conditions.

$$\mathcal{L}_A = \tilde{U}_A - \mu_A \sum_{k=1}^K k \left(P_{A,k}^{\text{ask}} P_{B,k}^{\text{grant}} - P_{B,k}^{\text{ask}} P_{A,k}^{\text{grant}} \right)$$

where μ_A is the Lagrange multiplier.

Starting with the partial derivative of the Lagrangian in terms of $\lambda_{A,1}$ and setting it equal to zero allows computing the value of the Lagrange multiplier $\mu_A = \lambda_{A,1}$. Setting the partial derivative of the Lagrangian with respect to $\lambda_{A,k} : k > 1$ equal to zero, and substituting the value of the Lagrange multiplier into the resulting equation gives

$$\lambda_{A,k} = k \lambda_{A,1}, \quad k > 1. \quad (6)$$

Next, starting from $\partial \mathcal{L}_A / \partial \theta_{A,1} = 0$ one can determine the threshold $\theta_{A,1}$ as a function of the thresholds $\lambda_{A,k}$

$$P_{B,1}^{\text{grant}} (\theta_{A,1} - \lambda_{A,1}) = \sum_{j=1}^K P_{B,j}^{\text{ask}} \int_0^{\lambda_{A,j}} (\lambda_{A,j} - l_j) f_{A,L_j}(l_j) dl_j. \quad (7)$$

Finally, setting $\partial \mathcal{L}_A / \partial \theta_{A,k} = 0, k > 1$ and using the solution for $\theta_{A,k-1}$, we end up with

$$\begin{aligned} \theta_{A,k} = \lambda_{A,k} & + \frac{P_{B,k-1}^{\text{grant}}}{P_{B,k}^{\text{grant}}} \left(\int_{\theta_{A,k-1}}^{\infty} (g_{k-1} - \lambda_{A,k-1}) f_{A,G_{k-1}}(g_{k-1}) dg_{k-1} \right. \\ & \left. + (\theta_{A,k-1} - \lambda_{A,k-1}) \int_0^{\theta_{A,k-1}} f_{A,G_{k-1}}(g_{k-1}) dg_{k-1} \right). \end{aligned} \quad (8)$$

The thresholds $\lambda_{A,k}, \theta_{A,k} \forall k$ that may maximize the Lagrangian must jointly satisfy equations (6)–(8) and also the constraint (3). Note that the above system of equations does not accept a closed-form solution but it is straightforward to solve numerically. Also, from equations (7,8) one can deduce that $\theta_{A,k} > \lambda_{A,k} \forall k$. In the Appendix, we show that the solution satisfying the first-order conditions and the constraint satisfies $\tilde{U}_A > 0$. Therefore, the proposed method can achieve better performance in comparison to the NE of the one-shot game. Finally, besides the calculation of the Lagrangian at the stationary point, we also compute it at the borders. The thresholds, either

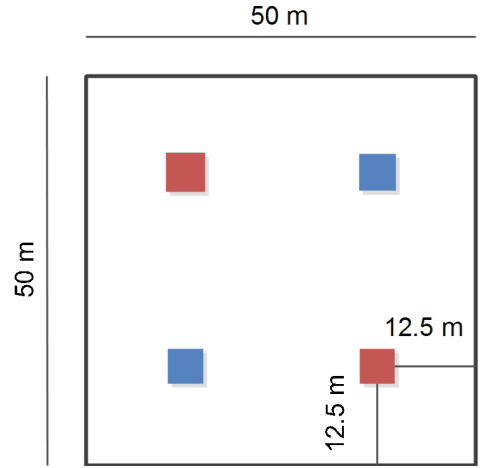


Fig. 1. Indoor inter-operator deployment scenario. Different colors represent BSs of different operators.

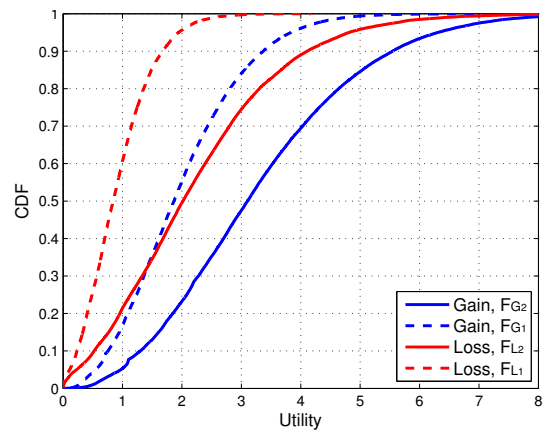


Fig. 2. Distribution of utility gains and utility losses for Operator *X* from spectrum usage favors over one and two CCs at the end of the initialization.

interior or border, maximizing the Lagrangian are selected.

IV. NUMERICAL EXAMPLES

In order to assess the performance of the proposed coordination protocol, we consider an indoor deployment scenario in a hall of a single-story building. The hall is a square with a side of 50 m. The BSs are partitioned into two groups as illustrated in Fig. 1 modeling a spectrum sharing scenario with two operators. The service areas of the operators fully overlap. A user is connected to the BS of its home network with the highest received signal level at its location. We consider a power law model for distance-based propagation pathloss with attenuation constant 10^{-4} and pathloss exponent 3.7. The available power budget on a CC is 20 dBm, the thermal noise power is -174 dBm/Hz and the noise figure is 10 dB. The SINR efficiency is $\gamma_{eff} = 2$. The bandwidth of a CC is $B = 20$ MHz. Initially, we consider that the LSP consists of two CCs.

First, we consider an initialization phase of 200 000 simulation snapshots (or equivalently 200 000 stage games). At each stage game, the user locations are independently generated according to the PPP and the operators calculate and keep track of their utility gains and utility losses from taking and granting favors over one and two CCs. We simulate many different network loads so that the distributions of utility gains and utility losses at the end of the initialization can be seen as

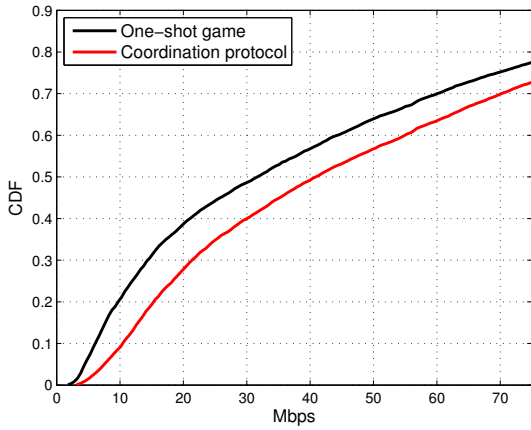


Fig. 3. Rate distribution for the users of Operator A obtained by the proposed scheme and by static spectrum allocation scheme where the operators always utilize the pool consisting of two CCs. Unequal mean network loads for the two operators.

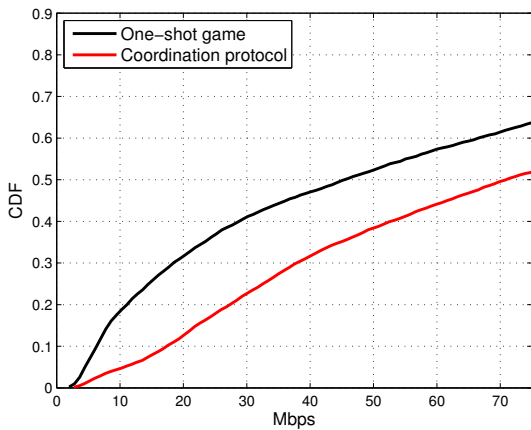


Fig. 4. Rate distribution for the users of Operator A obtained by the proposed scheme and by static spectrum allocation. Four CCs. Unequal mean network loads for the operators.

the steady state distributions over all possible network states. The different network loads are generated by varying the mean of the PPP used to model the locations of the users. In Fig. 2 we depict the distributions of utility gains and losses for an operator at the end of the initialization.

Next, we evaluate the performance in terms of the user rate distribution over a finite time horizon of 10 000 stage games following the initialization phase. Initially, the values of the thresholds are set arbitrarily equal to $\theta_{X,2} = 3$, $\theta_{X,1} = 1.5$, $\lambda_{X,2} = 2$ and $\lambda_{X,1} = 1$ for both operators. Every 100 stage games, the operators' probabilities for asking and granting favors are recomputed considering all stage games. Then, the decision thresholds are updated by solving the optimization problem (5) and so forth. Given the allocation of CCs at each stage of the game, the operators compute and keep track of the user rates. Recall that granted favors are valid only for a particular stage game. At the end of each stage game, the CC allocation returns to the default state i.e. both operators utilize all the CCs of the LSP. The performance of the proposed scheme is assessed in comparison with the NE of the one shot game, which is a static spectrum allocation scheme where both operators utilize all the CCs of the LSP.

First, we consider a scenario with network load asymmetry between the operators. The mean number of users for the first

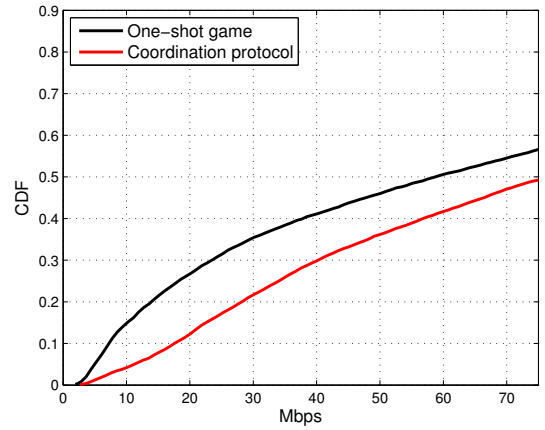


Fig. 5. Rate distribution for the users of Operator A obtained by the proposed scheme and also by static spectrum allocation. Four CCs. Equal mean network loads for the operators.

5 000 stage games are $N_A = 8$ and $N_B = 2$. In the second half of the simulation, the mean values are reversed. In Fig. 3, the rate distribution for the users of Operator A is depicted over the full course of the simulation. In the first 5 000 stage games, Operator B can mostly cope with fewer or no CCs due to the lower load and it grants more favors than the Operator A. In the second half of the simulation, Operator A returns the favors. Overall, Operator A offers better QoS in comparison with the QoS attained without any coordination e.g. it improves its mean user rate by approximately 28 %. The user rate distribution curves for Operator B follow the same trend and are not depicted. Fig. 4 depicts the rate distribution curves for the users of Operator A when the LSP consists of four CCs. One can see that the mean user rate increases by approximately 50 % while, the user rate at the 10 % of the distribution increases by more than 100 %.

Finally, we show that gains due to coordination can be achieved even in cases with equal mean network loads for the operators. In that case, the proposed protocol takes advantage of the instantaneous network load variations. In Fig. 5 we see that Operator A improves its mean user rate by approximately 25 % for mean number of users $N_A = N_B = 5$ and four CCs in the LSP.

V. CONCLUSIONS

In this paper, we considered co-primary spectrum sharing between two small cell operators deployed in the same geographical area. We considered a scenario where the operators have equal access rights on a spectrum pool and we proposed a protocol for coordinating the utilization of component carriers from the pool. According to it, an operator may ask for spectrum usage favors from its competitor. A spectrum usage favor means that the competitor would stop using some component carriers from the pool. An operator that has few users to serve could perhaps cope with less component carriers and grant the favor. Operators that have taken favors in the past are likely to return these favors in future and reciprocity is maintained. We formulated the interaction between the operators as a strategic, non-cooperative repeated game. Since it is hard to analyze the proposed game and find its Nash equilibrium, we resort to a heuristic strategy that uses a threshold-based test to decide whether to ask or grant a favor at each stage game. The decision thresholds depend on the current network realization and also in the history of previous interactions with

the competitor. We proved that the proposed strategy is strictly better as compared to the case without coordination between the operators. We illustrated that in an indoor deployment scenario, two operators are both able to offer higher user rates as compared to the case with no coordination, without revealing any operator-specific information to each other. Our results show that a rational operator, knowing that the opponent is rational and has a network with similar characteristics, has incentive to be cooperative. In future works, operators with non-similar load and network characteristics will be addressed, as well as models where the statistics of the underlying Poisson process changes to reflect e.g. variations in load due to time-of-day.

VI. APPENDIX

Using the fact that the decision thresholds over the distribution of utility losses are related as $\lambda_{A,k} = k\lambda_{A,1} \forall k : k > 1$, one can write equation (7) as

$$\sum_{k=1}^K P_{B,k}^{\text{ask}} \int_0^{\lambda_{A,k}} l_k f_{A,L_k}(l_k) dl_k = \lambda_{A,1} \sum_{k=1}^K k P_{B,k}^{\text{ask}} \int_0^{\lambda_{A,k}} f_{A,L_k}(l_k) dl_k - P_{B,1}^{\text{grant}} (\theta_{A,1} - \lambda_{A,1}). \quad (9)$$

Using equation (9) into equation (4), the excess utility can be read as

$$\begin{aligned} \tilde{U}_A = & \sum_{k=1}^K \left(P_{B,k}^{\text{grant}} \prod_{j=k+1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j \int_{\theta_{A,k}}^{\infty} g_k f_{A,G_k}(g_k) dg_k \right) \\ & + P_{B,1}^{\text{grant}} (\theta_{A,1} - \lambda_{A,1}) \prod_{j=1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j \\ & - \lambda_{A,1} \sum_{k=1}^K k P_{B,k}^{\text{ask}} \int_0^{\lambda_{A,k}} f_{A,L_k}(l_k) dl_k \prod_{j=1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j. \end{aligned} \quad (10)$$

According to the definition of the probabilities of granting a favor from equation (2), we note that the last term in equation (10) is equal to the right-hand side of the constraint in equation (3) scaled by $\lambda_{A,1}$. After replacing the last term of equation (10) by the left-hand side of the constraint in equation (3), we end up with

$$\begin{aligned} \tilde{U}_A = & \sum_{k=1}^K \left(P_{B,k}^{\text{grant}} \prod_{j=k+1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j \int_{\theta_{A,k}}^{\infty} g_k f_{A,G_k}(g_k) dg_k \right) \\ & + P_{B,1}^{\text{grant}} (\theta_{A,1} - \lambda_{A,1}) \prod_{j=1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j \\ & - \lambda_{A,1} \sum_{k=1}^K k P_{A,k}^{\text{ask}} P_{B,k}^{\text{grant}}. \end{aligned} \quad (11)$$

Using the probabilities of ask a favor from equation (1) into the last term of equation (11) and replacing back the decision thresholds, $\lambda_{A,k} = k\lambda_{A,1}$, the first and the last terms of equation (11) can be factorized together resulting to

$$\begin{aligned} \tilde{U}_A = & \sum_{k=1}^K \left(P_{B,k}^{\text{grant}} \prod_{j=k+1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j \int_{\theta_{A,k}}^{\infty} (g_k - \lambda_{A,k}) f_{A,G_k}(g_k) dg_k \right) \\ & + P_{B,1}^{\text{grant}} (\theta_{A,1} - \lambda_{A,1}) \prod_{j=1}^K \int_0^{\theta_{A,j}} f_{A,G_j}(g_j) dg_j. \end{aligned}$$

which is always positive since $\theta_{A,k} > \lambda_{A,k} \forall k$.

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